## 2021 James S. Rickards Fall Invitational

- 1. In order to solve this question we must first find out how long it takes Shikha and Srividya to pass each other. Since Shikha is driving at 50 miles per hour and Srividya is driving at 70 miles per hour and the total distance is 480 miles, after 4 hours Shikha will be 200 miles away from Rickards, and Srividya will be 280 miles from the beach. Since both of their distances total to 480 at the same time it is the time that they pass each other. The first mile marker at Rickards is mile marker 17, Shikha is 200 miles away from Rickards, and each mile marker is incremented by 10, so the mile marker they meet at would be 20 above the starting point for Shikha which is Rickards, making the final equation  $17 + 20 = \boxed{37}$ .
- 2. As a general rule whenever you are writing a repeating decimal in fraction form you should know that the numerator will always be the repeating part of the decimal and the denominator will be based on what the question is asking whether it be a single digit repeating or a double digit. Normally whenever there is a repeating decimal and you are asked to write it in fraction form their will be a 9, 99, or 999 in the denominator. In this special case since there is

a 0 after the decimal place before the repetition starts the answer would be  $\left| \frac{17}{990} \right|$ 

- 3. To solve this question we can set up an equation that will ease things up for us. We can consider the amount of people that were originally at Sruthi's party as S, and for Karina's party we can consider the amount of people there as K. To start we can make the equation S + 2 = 3(K 2) which represents what happens if 2 people leave Karina's party to go to Sruthi's party. The first step would be to expand the parentheses and make the equation S + 2 = 3K 6. Then we subtract 2 on both sides to get S = 3K 8. Since we know that the total amount of people present has to be between 8 and 16, if Karina had 5 people to start with at her party, then solving with the equation we can conclude that Sruthi had 7 people at her party originally making that a total of 12 people at both parties.
- 4. The prime factorization of a number is determined by finding factors of a number that are prime. In this case, 1001 is prime-factorized as (7)(11)(13), so the sum is 31. Now we find all the factors of 1001: 1, 7, 11, 13, 77, 91, 143, 1001. The sum is 1 + 7 + 11 + 13 + 77 + 91 + 143 + 1001 = 1344. Subtracting the prime factors we get 1344 31 = 1313. But we also must remember that 1 is not composite, so the actual sum is 1313 1 = 1312.
- 5. If we want to figure out how long it takes Nitish to complete his AOPS book, we must first calculate how much time it takes for him to complete  $\frac{1}{7}$  of a chapter. We can solve the expression  $\frac{20}{100} \cdot 60$ , which represents 20% of an hour, to find that it takes him 12 minutes to complete a seventh of a chapter. To complete a whole chapter it takes Nitish  $12 \cdot 7 = 84$  minutes. He has to complete  $2(2^3 + 3)$  chapters, or when simplified, 2(8 + 3) = 22 chapters. We can calculate how long it takes him in minutes by multiplying 22 chapters by 84 minutes per chapter giving us  $22 \cdot 84 = 1848$  minutes. 1848 minutes in seconds would simply be  $1848 \cdot 60$  which equals 110, 880 seconds.
- 6. The easiest way to solve this question would be to use the formula  $\frac{n!}{r!}$  which tells us all the ways to arrange a word. This formula interpreted in this question would be n - the amount of letters in the word ALGEBRA - which is 7, and then r which is the amount of the same letter being repeated which is A twice in this question so r would be 2. Solving this we get  $\frac{7!}{2!}$  which equals  $\frac{5040}{2}$  which equals  $\frac{2520}{2!}$ .
- 7. There are 3 ft in a yard, so there are 5280 ft in a mile. There are also 60 seconds in a minute. So we have  $(56390400)/(60 \cdot 5280) = 178$  miles per minute. To convert to scientific notation, we divide by 10 until the number is between 1 and 10. This means dividing by 100, so  $178 = 1.78 \times 10^2$ .
- 8. To answer this question you must know how many days there are in the months of May and June. In May there are 31 days and in June there are 30 days. Going by the fact that Melissa changes her profile picture every other day and starts on May 1st, she would also change her profile picture on these days in May: 3rd, 5th, 7th, 9th, 11th, 13th, 15th, 17th, 19th, 21st, 23rd, 25th, 27th, 29th, and 31st. Then in June she would change her profile picture on these days: 2nd, 4th, 6th, 8th, 10th, 12th, 14th, 16th, 18th, 20th, 22nd, 24th, 26th, 28th, and 30th. The total amount of days she changes her profile picture in the months of May and June is 31.
- 9. To solve this question we must start by finding out how much the total cost is for Shrung to get his name changed. The flat fee of \$80 plus \$17 per extra letter would give us a total of \$318. If he already has \$40, then he only needs to earn \$318 \$40 = 278. That means he must work 28 hours to get enough money to pay for his official name change. If he can only work 3 hours a day and he starts on a Wednesday and only works on weekdays he will have enough money after 10 days which would be Tuesday.

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- 10. Considering Karthik's pace if he drinks  $\frac{1}{4}$  pint of smoothie in  $\frac{1}{4}$  minute, he can drink 1 pint in 1 minute. Considering Annika's pace, 6 fluid ounces in 24 seconds equals 15 fluid ounces in 60 seconds or 1 minute. Therefore, Karthik will finish his drink first, but by how much time will he beat Annika? If they have to drink 3 gallons that means they have to drink 12 quarts which means they have to drink 24 pints each or 384 pints. It will take Karthik 24 minutes, and it will take Annika  $\frac{384}{15}$  minutes or 25.6 minutes. Karthik drinks his smoothie 1.6 minutes before Annika.
- 11. In order to solve this question we must create a system of equations as follows, m standing for McChickens and b standing for Big Macs:

$$21m + 16b = 67$$
$$19m + 26b = 87.70$$

By multiplying the top equation by 19 and the bottom equation by 21 we can eliminate the m value and find the value of b

$$399m + 304b = 1273$$
  
 $399m + 546b = 1841.7$ 

By subtracting the equations above we get

$$-242b = -568.7$$
  

$$b = \frac{-568.7}{-242}$$
  

$$b = 2.35$$

Now that we know the value of b which is the value of a Big Mac we can substitute back into the first equation and find the value of m which is the value of a McChicken

$$21m + 16(2.35) = 67$$
  

$$21m + 37.6 = 67$$
  

$$21m = 67 - 37.6$$
  

$$21m = 29.4$$
  

$$m = \frac{29.4}{21}$$
  

$$m = 1.4$$

Now that we know the cost of m we can add the two costs to find the total cost of one Big Mac and one McChicken to be 2.35 + 1.40 = 3.75.

12. In the quote "I love doing math everyday" there are 9 vowels and 13 consonants. That equals  $(9 \cdot 2) + (13 \cdot 1)$  which is 31. Eric's answer is found by solving the expression  $\frac{(5! \cdot 2)}{5}$  which equals  $\frac{(120 \cdot 2)}{5}$ . Next, we can simplify the top and equate the expression to  $\frac{240}{5}$ . Finally by solving  $\frac{240}{5}$  we get Eric's value as 48. The question asks how far off Eric is from the actual value which is 31, so we do 48 - 31 to get [17].

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13. Let a be the amount of Solution A put into the glass, and b be the amount of Solution B put into the glass. Since the two volumes must add to 3.5 liters, we have a + b = 3.5. The volume of nether wart in a is 0.1a because Solution A is 10% nether wart, and by the same logic the volume of nether wart in b is 0.24b. The two volumes of nether warts must add to 0.12(3.5) because the 3.5-liter solution is 12% nether wart. Thus we have 0.1a + 0.24b = 0.12(3.5).

We solve the system of equations created by a + b = 3.5 and 0.1a + 0.24b = 0.12(3.5). To do this, we multiply the first equation by 0.1, then subtract it from the second to get 0.14b = 0.02(3.5) = 0.07 so that b = 0.5 liters. Thus, plugging into the first equation we find that a = 3.5 - b = 3 liters.

Since the original beakers were each 5 liters, there are 2 liters of Solution A left, and 4.5 liters of Solution B left. Mixing these forms a solution of volume 6.5 liters, and the volume of nether wart in this solution is 2(0.1) + 4.5(0.24) = 1.28 liters. The concentration is the ratio of these, which is  $\frac{128}{650} = \boxed{\frac{64}{325}}$ 

14. To solve this question we must simply consider the value we need to find, which is how many people like all three series or neither of them, as x. In doing so we can create three expressions to find x:

$$107 - (51 - x + x + 65 - x)$$
  

$$93 - (38 - x + x + 65 - x)$$
  

$$76 - (38 - x + x + 51 - x)$$

The expressions above can be explained as this: the first expression represents all the individuals who don't like the Percy Jackson Series and don't like both Percy Jackson and Harry Potter and don't like both Percy Jackson and Shadowhunters. The second expression represents all the individuals who don't like the Harry Potter series and don't like both Harry Potter and Shadowhunters and don't like Harry Potter and Percy Jackson. The third expression represents all the individuals who don't like both Shadowhunters and Harry Potter and On't like both Shadowhunters and Harry Potter and On't like both Shadowhunters and Harry Potter and On't like both Shadowhunters and Percy Jackson. Simplifying these expressions we get

$$\begin{array}{r}
107 - (116 - x) \\
93 - (103 - x) \\
76 - (89 - x)
\end{array}$$

Finally we get

$$\begin{array}{c} x - 9 \\ x - 10 \\ x - 13 \end{array}$$

Using these values we can make one final equation

$$(x-13) + (38 - x) + (51 - x) + (65 - x) + (x - 10) + (x - 9) = x + 122$$
  
150 = x + 122 x = 28

The number of people who like all three series or neither of the series is 28.

15. To solve this question we can use a system of equations as follows:

 $0.08x + 0.06y = 7250 \ 0.06(x + y = 100000)$ 

The first equation in the system of equations represents Siri's 8% investment interest in addition to her 6% investment interest per year equating to \$7,250. The second equation shows how much 6% of her total investment gives. Solving the system of equations gives us the value of her 8% investment interest

 $0.08x + 0.06y = 7250 \ 0.06x + 0.06y = 6000) \ 0.02x = 1250 \ x = 62500$ 

Since we solved for x, we found that Siri's 8% investment interest equates to \$62,500